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NINTH PROGRESS REPORT

on the

DEVELOPMENT OF A DIRIGIBLE BOMB

SECRET

Contract No. OEM sr 240

April 30, 1943.

Contractor: Division of Industrial Cooperation
Massachusetts Institute of Technology
Cambridge, Massachusetts.

[redacted]

DIRIGIBLE BOMB TRAJECTORIES

SECRET

DIRIGIBLE BOMB TRAJECTORIES

I. METHOD OF COMPUTING

General Equations

The equations will be set up in terms of a right-handed system of coordinates X, Y, Z, such that + X is the direction of the horizontal component of initial velocity, + Y is the lateral direction to the right as viewed from above, and + Z is the downward direction. The pitch angle Θ is positive for the bomb axis B pointing above or forward of the velocity direction V (sailing). The yaw angle ψ is positive for B pointing to the right of V as viewed from above. The roll angle ϕ is the angle of rotation around the longitudinal axis, taken clockwise when looking upwind. The relations involved are illustrated in Fig. 1.

The terminology of the cruciform radial fin type bomb will be used throughout this discussion, although the equations apply equally well to other models such as those with cylindrical shrouds. We use the terminology of the cruciform model merely because the horizontal and vertical fins provide convenient reference planes. As shown in Fig. 1, the pitch angle Θ, and the yaw angle ψ are both positive. The plane BOA is in the plane of the vertical fins, and the plane VOA is perpendicular to the vertical fins.

In controlled flight, the bomb is acted upon by

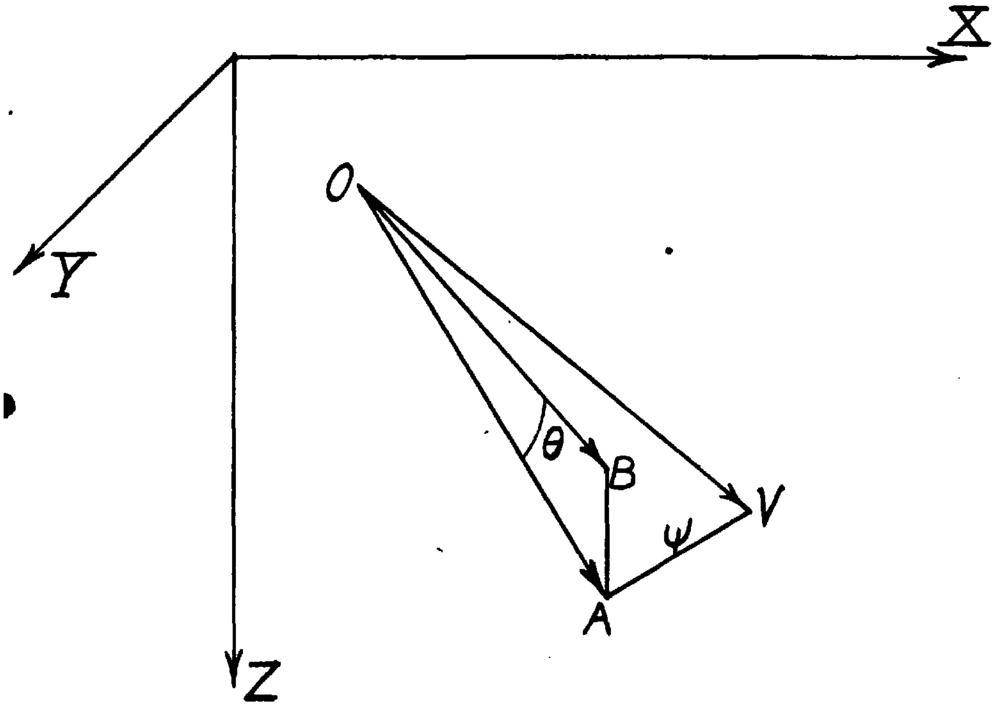

Fig 1. System of Coordinates.

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two forces: a pull down equal to the weight, and an aerodynamic force which can be resolved into three components: a drag D which is opposite in direction to V, a lift force L perpendicular to V and in the plane of the vertical fins, and a side force S which is perpendicular to V and in the plane of the horizontal fins. Positive L is in the upward forward direction, and positive S is toward the right. It is assumed that the air is motionless with respect to the ground so that the velocity V of the bomb is also its velocity with respect to the air. The equations of motion are:

$$
\begin{aligned}
m\frac{d^2x}{dt^2} &= -D\left(\frac{V_x}{V}\right) + L\left(\frac{L_x}{L}\right) + S\left(\frac{S_x}{S}\right) \\
m\frac{d^2y}{dt^2} &= -D\left(\frac{V_y}{V}\right) + L\left(\frac{L_y}{L}\right) + S\left(\frac{S_y}{S}\right) \qquad (1) \\
m\frac{d^2z}{dt^2} &= mg - D\left(\frac{V_z}{V}\right) + L\left(\frac{L_z}{L}\right) + S\left(\frac{S_z}{S}\right)
\end{aligned}
$$

Set

$$
\begin{aligned}
D &= C_D'V^2 \\
L &= C_L'V^2 \qquad (2) \\
S &= C_S'V^2
\end{aligned}
$$

Primes are used on the C coefficients to distinguish them from the true lift and drag coefficients.

$$
\begin{aligned}
\frac{d^2x}{dt^2} &= -\frac{C_D'}{m}VV_x + \frac{C_L'}{m}V^2\left(\frac{L_x}{L}\right) + \frac{C_S'}{m}V^2\left(\frac{S_x}{S}\right) \\
\frac{d^2y}{dt^2} &= -\frac{C_D'}{m}VV_y + \frac{C_L'}{m}V^2\left(\frac{L_y}{L}\right) + \frac{C_S'}{m}V^2\left(\frac{S_y}{S}\right) \qquad (3) \\
\frac{d^2z}{dt^2} &= g - \frac{C_D'}{m}VV_z + \frac{C_L'}{m}V^2\left(\frac{L_z}{L}\right) + \frac{C_S'}{m}V^2\left(\frac{S_z}{S}\right)
\end{aligned}
$$

[redacted]

Set

$$
\begin{aligned}
A &= \frac{C'_D}{m} V V_x \\
B &= \frac{C'_L}{m} V^2 \left(\frac{L_x}{L}\right) \\
C &= -\frac{C'_S}{m} V^2 \left(\frac{S_x}{S}\right) \\
D &= \frac{C'_D}{m} V V_y \\
E &= \frac{C'_L}{m} V^2 \left(\frac{L_y}{L}\right) \\
F &= \frac{C'_S}{m} V^2 \left(\frac{S_y}{S}\right) \\
G &= \frac{C'_D}{m} V V_z \\
H &= -\frac{C'_L}{m} V^2 \left(\frac{L_z}{L}\right) \\
I &= -\frac{C'_S}{m} V^2 \left(\frac{S_z}{S}\right)
\end{aligned}
\qquad (4)
$$

Minus signs are introduced in the definitions (4) in such a way as to minimize the number of minus signs appearing in the final computation.

$$
\begin{aligned}
\frac{dV_x}{dt} &= -A + B - C & \Delta V_x &= (\overline{-A+B-C})\,\Delta t \\
\frac{dV_y}{dt} &= -D + E + F & \Delta V_y &= (\overline{-D+E+F})\,\Delta t \\
\frac{dV_z}{dt} &= g - G - H - I & \Delta V_z &= (g - \overline{G-H-I})\,\Delta t
\end{aligned}
\qquad (5)
$$

The average sign means the average value during the time interval Δt. It is convenient to let $\Delta t = 1.0$ sec.

$$
\begin{aligned}
V_{x,t} &= V_{x,t-1} + (\overline{-A+B-C})^{t-1,t} \\
V_{y,t} &= V_{y,t-1} + (\overline{-D+E+F})^{t-1,t} \\
V_{z,t} &= V_{z,t-1} + 32.2 + (\overline{-G-H-I})^{t-1,t}
\end{aligned}
\qquad (6)
$$

[redacted]

$$X_t = X_{t-1} + V_{x,t-1} + \tfrac{1}{2}\left(\overline{-A+B-C}\right)^{t-1,t}$$
$$Y_t = Y_{t-1} + V_{y,t-1} + \tfrac{1}{2}\left(\overline{-D+E+F}\right)^{t-1,t} \qquad (7)$$
$$Z_t = Z_{t-1} + V_{z,t-1} + 16.1 + \tfrac{1}{2}\left(\overline{-G-H-I}\right)^{t-1,t}$$

The coefficients $\frac{C_D}{m}$, $\frac{C_L}{m}$, $\frac{C_S}{m}$ are computed from wind tunnel measurements and the weight of bomb and plotted for a series of pitch and yaw angles as a function of altitude. The quantities V_x, V_y, V_z and V are at each step in the calculation known for the previous instant t-1. The ratios $(\frac{L_x}{L})$, $(\frac{S_x}{S})$, $(\frac{L_y}{L})$, etc., are determined by θ, ψ, V_x, V_y, V_z and the roll angle restrictions, for example the gyro restrictions if the roll angle is maintained by a directional gyro.

Case I. Two Dimensional Trajectory in the XZ Plane. Range Variation Only.

It is assumed that the initial velocity is in the XZ plane, the rudder angle $\delta_r = 0$, and the gyro maintains the vertical fin in the XZ plane. Equations (6) and (7) become:

$$V_{x,t} = V_{x,t-1} + \left(\overline{B-A}\right)^{t-1,t}$$
$$V_{z,t} = V_{z,t-1} + 32.2 - \left(\overline{G+H}\right)^{t-1,t} \qquad (9)$$

$$X_t = X_{t-1} + V_{x,t-1} + \tfrac{1}{2}\left(\overline{B-A}\right)^{t-1,t}$$
$$Z_t = Z_{t-1} + V_{z,t-1} + 16.1 - \tfrac{1}{2}\left(\overline{G+H}\right)^{t-1,t} \qquad (10)$$

Since for this case the lift force vector L lies in the XZ plane perpendicular to V

$$
\begin{aligned}
A &= \frac{C_D'}{m} V V_x \\
B &= \frac{C_L'}{m} V^2 \left(\frac{L_x}{L}\right) = \frac{C_L'}{m} V V_z \\
C &= \frac{C_D'}{m} V V_z \\
E &= -\frac{C_L'}{m} V^2 \left(\frac{L_z}{L}\right) = +\frac{C_L'}{m} V V_x
\end{aligned}
\qquad (11)
$$

A schedule for the "range only" type of calculation is illustrated by the calculations for the Gulf bomb No. 1 Eglin Field, Dec., 1942. Long horizontal (type B), short vertical (type C) fins as of the Oct. 3, 4, 1942, wind tunnel measurements. Weight 1000 lb., altitude 15,000 ft., plane speed 220 ft. per sec., $\delta_R = 0$. Elevator settings 0 - 22 sec. $\delta_E = -10°$; 22 - 26 sec. $\delta_E = 0$; 26 - end $\delta_E = -10°$. The wind tunnel measurements of Oct. 3, 4 are available in preliminary form in the Gulf Progress Report of Oct. 15, 1942 and in better final form in the M. I. T. Progress Report No. 7. From the curve labelled pitch of Fig. 27 Gulf Report, or from the Fin B curve of Fig. 16 M. I. T. Report, an average value of pitch angle $\theta = -10.6°$ is obtained for $\delta_E = -10°$. The next step is to construct the ballistic coefficient curves $\frac{C_L'}{m}$ and $\frac{C_D'}{m}$ as a function of altitude. The lift and drag forces are given in terms of the true lift and drag coefficients by the relations

$$
\begin{aligned}
L &= C_L\, S\, \frac{\rho V^2}{2} \\
D &= C_D\, S\, \frac{\rho V^2}{2}
\end{aligned}
\qquad (12)
$$

where $S = 1.868$ sq. ft. is the cross sectional area of the bomb, and ρ is the air density (standard value taken

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as ρ_o = .002378 slugs per cu.ft.). From equations (2)

$$L = C'_L V^2$$

$$D = C'_D V^2$$

Combining (12) and (2)

$$\frac{C'_L}{m} = \frac{C_L}{m} S \frac{\rho}{2}$$

$$\frac{C'_D}{m} = \frac{C_D}{m} S \frac{\rho}{2} \qquad (13)$$

From the Gulf Report Fig. 28, or the M. I. T. Report Fig. 17:

$\theta = 0$	$C_L = 0$	$C_D = 0.205$
$\theta = -10.6°$	$C_L = -0.640$	$C_D = 0.290$

Hence for $\theta = -10.6°$ and standard pressure

$$\frac{C'_L}{m} = -0.640 \times \frac{32.2}{1000} \times 1.868 \times \frac{.002378}{2} = -0.457 \times 10^{-4}.$$

To obtain $\frac{C'_L}{m}$ at any desired altitude, the value 0.457×10^{-4} is multiplied by the ratio $\frac{\rho}{\rho_o}$ for the desired altitude. Table I illustrates this for $\frac{\rho}{\rho_o}$ values corresponding to a ground temperature of 77°F, and a pitch angle of 10.6°.

Table I. Computation of Ballistic Coefficients.

Altitude	$\frac{\rho}{\rho_o}$ (77°F)	$\frac{C'_L}{m} \times 10^4$
0 ft.	0.959	0.438
3,000	0.882	0.403
6,000	0.809	0.370
9,000	0.740	0.338
12,000	0.676	0.309
14,000	0.636	0.291
16,000	0.596	0.273
20,000	0.526	0.242
25,000	0.448	0.205

Fig. 2 gives the three ballistic coefficient curves which are necessary for the calculation. The schedule for this "range only" type of calculation is illustrated in Table II. Each of the quantities $V_{x,t}$, $V_{z,t}$, X_t, Z_t is given by equations 9, 10 in terms of the quantities for t-1 and the average coefficients $(\overline{B-A})$ and $(\overline{G+H})$. These coefficients are the average values between t-1 and t and are obtained by extrapolation from the quantities in the last two columns for times t-1 and t-2. Where there is no discontinuous change in elevator setting

$$(\overline{B-A})^{t-1,t} = (B-A)_{t-1} + \frac{1}{2}\left\{(B-A)_{t-1} - (B-A)_{t-2}\right\}$$

To start the calculation, the values of X, Z, V_x, V_z are all known for t = 0 and can be filled in. From the altitude column (15,000 - Z), and the elevator setting, the ballistic coefficients are read off the curves of Fig. 2. In the last two columns, we obtain (B-A) and (G+H) values for t = 0. To start the t = 1 line, we use as $(\overline{B-A})$ and $(\overline{G+H})$ the t = 0 values for (B-A) and (G+H). For the t = 2 line, we already have t = 0 and t = 1 values of the coefficients and from here on can put in the correct extrapolated average values. It should be noted that the first thirteen columns give the desired positions and velocities at the time t. The remaining columns are to get the (B-A) and (G+H) coefficients at the time t in preparation for the t+1 calculation.

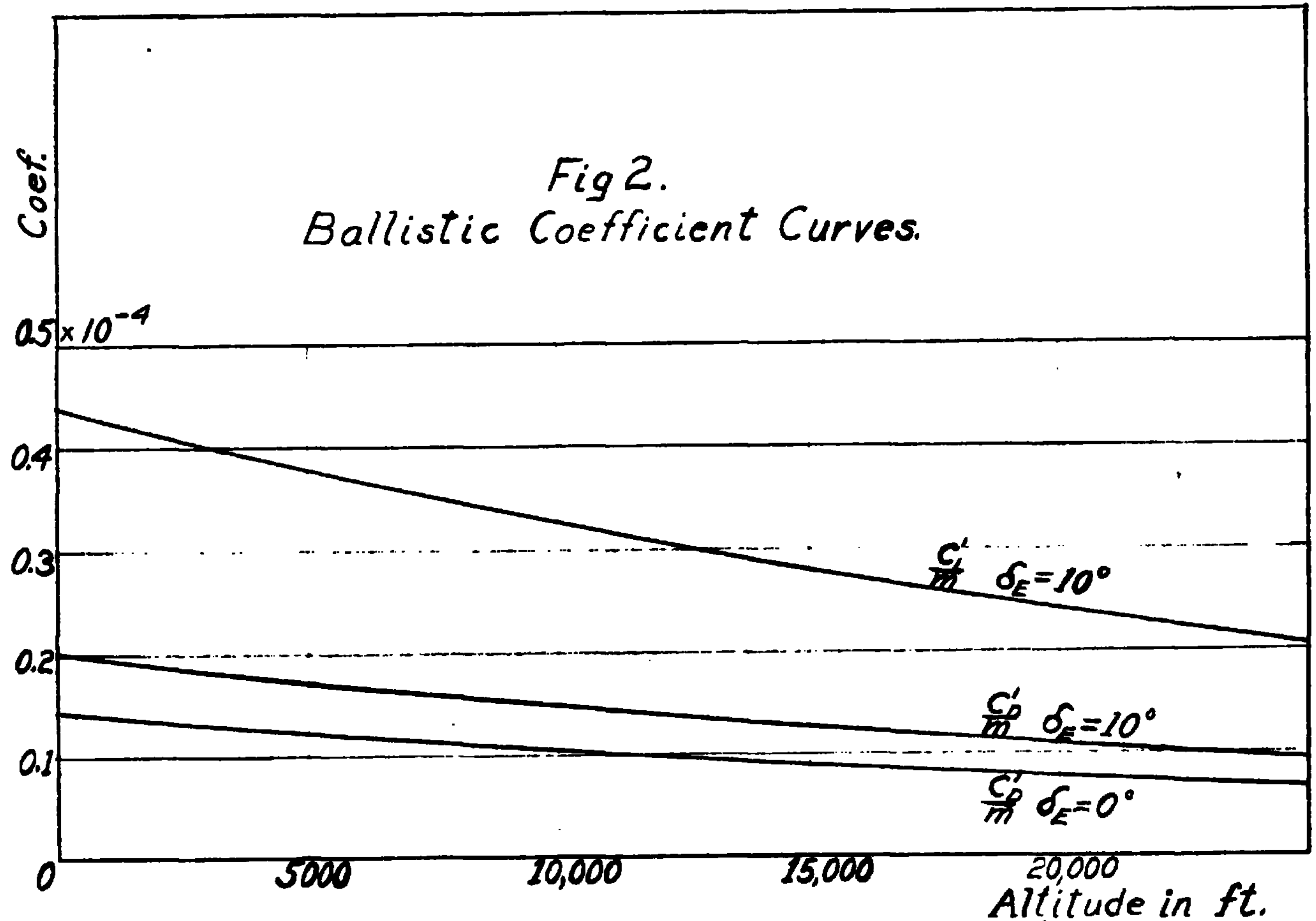

Fig 2.
Ballistic Coefficient Curves.

TABLE II "RANGE ONLY"

t_{sec}	δ_E	$\overline{B-A}$	$\overline{G+H}$	X_t	Z_t	ΔX	ΔZ	$\Delta^2 Z$	$15000-Z$	V_x	V_z	V	$\frac{C_L'}{m}$	VV_z	VV_x	$\frac{C_D'}{m}$	B $\frac{C_L'}{m}VV_z$	H $\frac{C_L'}{m}VV_x$	A $\frac{C_D'}{m}VV_x$	G $\frac{C_D'}{m}VV_z$	$B-A$	$G+H$
0	-10°	--	—	0	0	—	—	—	15000	220.0	0.0	220	-.282	0.0	4.84	+.128	0.0	-1.37	+0.62	0.0	-.62	-1.37
1	-10°	-.62	-1.37	219.7	16.8	219.7	16.8	—	14983	219.4	33.6	222	-.282	0.74	4.87	.128	-0.21	-1.37	0.62	+0.09	-.83	-1.28
2	-10°	-.94	-1.24	438.5	67.1	218.8	50.3	33.5	14933	218.5	67.0	229	-.283	1.53	5.01	.128	-0.43	-1.41	0.64	0.20	-1.07	-1.21
3	-10°	-1.19	-1.17	656.4	150.8	217.9	83.7	33.4	14849	217.3	100.4	240	-.283	2.40	5.21	.129	-0.68	-1.47	0.67	0.31	-1.35	-1.16
4	-10°	-1.49	-1.13																			
20	-10°	-14.9	+3.88	3703	6596	108	630	29	8404	101.3	645.0	653	-.344	42.1	6.60	+.157	-14.50	-2.27	1.04	6.60	-15.55	+4.33
21	-10°	-16.4	4.80	3796	7255	93	659	29	7745	84.8	672.9	678	-.350	45.6	5.75	.160	-15.60	-2.05	0.92	7.30	-16.52	5.25
22	0	-17.0	5.70	3872	7941	76	686	27	7059	67.8	698.9	702	.000	49.6	4.88	.125	0.00	0.00	0.61	6.20	-0.61	6.20
23	0	-0.6	6.70	3940	8653	68	712	26	6347	67.2	724.5	728	.000	52.9	4.36	.128	0.00	0.00	0.56	6.76	-0.56	6.76
24	0	-0.5	7.04	4007	9390	67	737	25	5610	66.7	749.7	752	.000	56.3	4.46	.130	0.00	0.00	0.58	7.32	-0.58	7.32
25	0	-0.6	7.60	4073	10152	66	762	25	4848	66.1	774.3	776	.000	60.0	4.55	.133	0.00	0.00	0.61	8.00	-0.61	8.00
26	-10°	-0.6	8.34	4139	10938	64	786	24	4062	65.5	798.2	800	-.390	63.9	4.65	.175	-24.94	-1.82	0.79	11.18	-25.73	9.36
27	-10°	-25.7	10.04	4192	11747	53	809	23	3253	39.8	820.4	822	-.400	67.5	3.74	.180	-26.60	-1.50	0.66	12.00	-27.26	10.50

TABLE III "AZIMUTH ONLY"

t_{sec}	δ_R	$\overline{A+C}$	$\overline{F-D}$	$\overline{G+I}$	X	Y	Z	$20{,}000-Z$	ΔX	ΔY	ΔZ	$\Delta^2 Z$	V_x	V_y	V_z	$\sqrt{V_x^2+V_z^2}$	V	a VV_x	b VV_y	c VV_z	d $V\sqrt{\ }$	e $\frac{V_y}{V}$	$\frac{C_D'}{m}$	$\frac{C_s'}{m}$	A $\frac{C_D'}{m}a$	C $\frac{C_s'}{m}ae$	D $\frac{C_D'}{m}b$	F $\frac{C_s'}{m}d$	G $\frac{C_D'}{m}c$	I $\frac{C_s'}{m}ce$	$A+C$	$F-D$	$G+I$
0	0	—	—	—	0.0	—	0.0	20,000	—	—	—	—	220.0	—	0.0	—	220.0	4.84	—	0.0	—	—	.075	.00	.36	—	—	—	.00	—	0.36	—	0.00
1	0	.36	—	.00	219.8	—	16.1	19984	219.8	—	16.1	—	219.6	—	32.2	—	222.0	4.86	—	0.71	—	—	.075	.00	.36	—	—	—	0.05	—	0.36	—	0.05
2	0	.36	—	.07	439.2	—	64.4	19936	219.4	—	48.3	32.2	219.2	—	64.4	—	228.2	5.00	—	1.47	—	—	.075	.00	.38	—	—	—	0.11	—	0.38	—	0.11
3	0	.39	—	.14	658.2	—	144.8	19855	219.0	—	80.4	32.1	218.8	—	96.5	—	239.0	5.23	—	2.31	—	—	.077	.00	.40	—	—	—	0.18	—	0.40	—	0.18
4	0	.41	—	.22																													
13	0	.78	—	1.49	2821	—	2694	17306	214	—	396	31	213.1	—	410.9	—	464	9.86	—	19.0		—	.082	—	.81	—	—	—	1.56	—	0.81	—	1.56
14	0	.84	—	1.66	3034	—	3120	16880	213	—	426	30	212.3	—	441.4	—	491	10.42	—	21.7	—	—	.083	—	.86	—	—	—	1.80	—	0.86	—	1.80
15	20°	.89	—	1.92	3246	—	3576	16423	212	—	457	30	211.4	—	471.7	518	518	10.92	0.0	24.4	26.8	.00	.200	.450	2.18	0.00	0.00	12.05	4.88	0.0	2.18	12.05	4.88
16	20°	2.18	12.05	4.88	3456	6.0	4062	15938	210	6.0	485	28	209.2	12.1	499.0	541	541	11.30	0.65	27.0	29.2	.022	.200	.450	2.26	0.11	0.13	13.13	5.40	0.27	2.37	13.00	5.67
17	20°	2.47	13.50	6.07	3664	24.9	4574	15426	208	18.9	512	27	206.7	25.6	525.1	564	564	11.70	1.45	29.6	31.8	.04[illegible]	.200	.455	2.34	0.24	0.29	14.46	5.92	0.61	2.58	14.17	6.53
18	20°	2.68	14.75	6.96																													

[redacted]

At t = 22.0 sec., the elevator changes from $\delta_E = -10^\circ$ to $\delta_E = 0^\circ$. The first thirteen columns are filled in with $(\overline{B-A})$ and $(\overline{G+H})$ coefficients obtained from the t = 21 and t = 20 values. The remaining columns in the t = 22 line are filled in with the ballistic coefficients for the new setting, since the purpose of these columns is to get (B-A) and (G+H) values in preparation for the t = 23 calculation. To start the t = 23 line, put in for $(\overline{B-A})$ and $(\overline{G+H})$ the values of (B-A) and (G+H) computed at t = 22. At the end of the t = 23 line, we have coefficients for t = 23. If the average of the t = 22 and t = 23 values differ appreciably from the approximate values which have been used, put in the correct values of $(\overline{B-A})$ and $(\overline{G+H})$ and do the line again. For a check on the computations, first differences in X and second differences in Z are tabulated. In the tabulation, a factor of 10^{-4} is left out in the $\frac{C_L}{m}$ and the $\frac{C_D}{m}$ columns, and a factor of 10^4 in the VV_x and VV_z columns, since these factors will cancel out in the final B, H, A, and G columns. $\frac{C_D}{m}$ is always positive, $\frac{C_L}{m}$ will be positive or negative for positive or negative pitch angles Θ. The significance of the $(\overline{B-A})$ and $(\overline{G+H})$ coefficients is probably obvious, they are average X and Z components of acceleration of the bomb. The complete trajectory for the calculation illustrated by Table II is given by Curve 1 of Fig. 4.

Case II. Azimuth Control Only.

The trajectory for this case is necessarily three-dimensional. The elevator angle $\delta_E = 0$ and hence $C'_L = 0$, and B, E, and H are all zero. Equations (6) and (7) become:

$$\left.\begin{aligned} V_{x,t} &= V_{x,t-1} - \overline{(+A+C)}^{\,t-1,t} \\ V_{y,t} &= V_{y,t-1} + \overline{(-D+F)}^{\,t-1,t} \\ V_{z,t} &= V_{z,t-1} + 32.2 - \overline{(+G+I)}^{\,t-1,t} \end{aligned}\right\} \qquad (14)$$

$$\left.\begin{aligned} X_t &= X_{t-1} + V_{x,t-1} - \tfrac{1}{2}\,\overline{(+A+C)}^{\,t-1,t} \\ Y_t &= Y_{t-1} + V_{y,t-1} + \tfrac{1}{2}\,\overline{(-D+F)}^{\,t-1,t} \\ Z_t &= Z_{t-1} + V_{z,t-1} + 16.1 - \tfrac{1}{2}\,\overline{(+G+I)}^{\,t-1,t} \end{aligned}\right\} \qquad (15)$$

where,

$$A = \frac{C'_D}{m} VV_x$$

$$C = -\frac{C'_S}{m} V^2\left(\frac{S_x}{S}\right)$$

$$D = \frac{C'_D}{m} VV_y$$

$$F = \frac{C'_S}{m} V^2\left(\frac{S_y}{S}\right)$$

$$G = \frac{C'_D}{m} VV_z$$

$$I = -\frac{C'_S}{m} V^2\left(\frac{S_z}{S}\right)$$

The ratios $\left(\frac{S_x}{S}\right)$, $\left(\frac{S_y}{S}\right)$, $\left(\frac{S_z}{S}\right)$ will depend upon the particular kind of roll control which is used. The equations will be developed for the gyro control used by the M.I.T. and Gulf high angle dirigible bombs. The outer gimbal of the directional gyro rotates about the longitudinal axis, and carries a contact arm which operates the ailerons in such a way as to keep the plane of the vertical fins in the plane of the outer

[redacted]

gimbal. The wheel of the directional gyro was initially rotating in the XZ plane and will remain in this plane. The axis of rotation of the inner gimbal lies in the plane of the wheel and hence remains in the XZ plane. Since the outer gimbal is in the plane of the vertical fins, the axis of the inner gimbal will be perpendicular to the plane of the horizontal fins. The plane of the horizontal fins is thus perpendicular to a line in the XZ plane, and therefore perpendicular to the XZ plane. This simple relation is perfectly general, and applies even with combined pitch and yaw. The relation is so fundamental to the analysis of gyro operation that we shall name the plane of the horizontal fins the "Reference Plane." The "Reference Plane" remains perpendicular to the XZ plane, or stated in another way, the "Reference Plane" intersects the ground on a line parallel to the Y axis.

With rudder control only (yaw only) the velocity direction remains in the plane of the horizontal fins, and hence the bomb vector B, the velocity vector V and the side force vector S all lie in the reference plane with S perpendicular to V. The relation is shown in Fig. 3 for a positive yaw angle ψ.

$$S_x = - S \sin\varepsilon \cos\delta$$

$$S_y = S \cos\varepsilon$$

$$S_z = - S \sin\varepsilon \sin\delta$$

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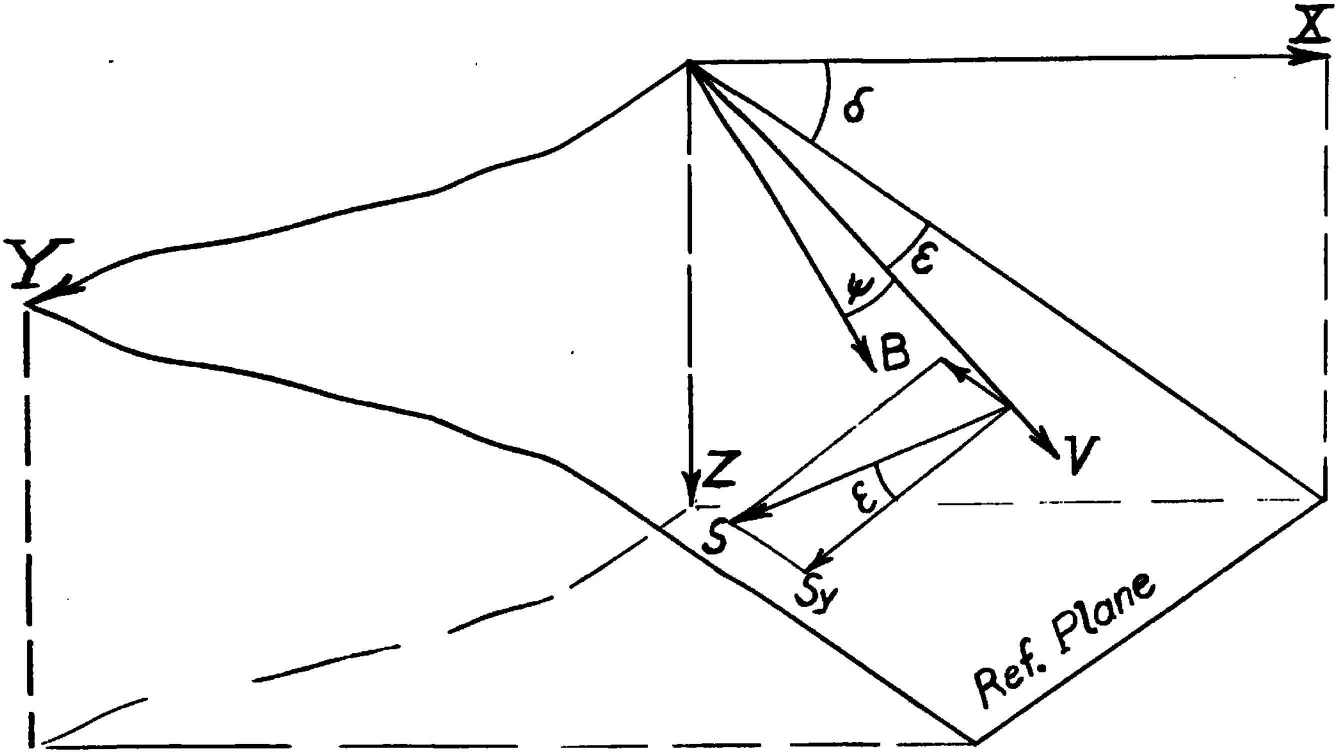

Relations for Yaw only

Fig. 3.

[redacted]

$$\left.\begin{aligned}
\frac{S_x}{S} &= -\frac{V_y}{V}\,\frac{V_x}{\sqrt{V_x^2 + V_z^2}} \\
\frac{S_y}{S} &= \frac{\sqrt{V_x^2 + V_z^2}}{V} \\
\frac{S_z}{S} &= -\frac{V_y}{V}\,\frac{V_z}{\sqrt{V_x^2 + V_z^2}}
\end{aligned}\right\} \qquad (16a)$$

$$\left.\begin{aligned}
A &= \frac{C'_D}{m}\,VV_x \\
C &= \frac{C'_S}{m}\,VV_y\,\frac{V_x}{\sqrt{V_x^2 + V_z^2}} \\
D &= \frac{C'_D}{m}\,VV_y \\
F &= \frac{C'_S}{m}\,V\,\sqrt{V_x^2 + V_z^2} \\
G &= \frac{C'_D}{m}\,VV_z \\
I &= \frac{C'_S}{m}\,VV_y\,\frac{V_z}{\sqrt{V_x^2 + V_z^2}}
\end{aligned}\right\} \qquad (16b)$$

Let

$$\left.\begin{aligned}
a &= VV_x \\
b &= VV_y \\
c &= VV_z \\
d &= V\sqrt{V_x^2 + V_z^2} \\
e &= \frac{V_y}{\sqrt{V_x^2 + V_z^2}}
\end{aligned}\right\} \qquad (16)$$

Then

$$\begin{aligned}
A &= \frac{C'_D}{m}\,a \\
C &= \frac{C'_S}{m}\,ae \\
D &= \frac{C'_D}{m}\,b \\
F &= \frac{C'_S}{m}\,d \\
G &= \frac{C'_D}{m}\,c \\
I &= \frac{C'_S}{m}\,ce
\end{aligned} \qquad (17)$$

For the "yaw only" type of trajectory the equations of motion are given by (14) (15) with the coefficients defined by equations (16) and (17). In a "yaw only" trajectory with roll orientation maintained by a directional gyro as explained above, the inner gimbal will not necessarily remain perpendicular to the outer gimbal. The angle μ through which the inner gimbal rotates from the perpendicular position can be seen from Fig. 3 to be

$$\mu = \psi + \mathcal{E} = \psi + \tan^{-1} e \qquad (18)$$

It is desirable that μ be kept small, say $\mu < 45^\circ$, since the effective component of angular momentum of the gyro approaches zero for μ approaching 90°.

The schedule for a "yaw only" calculation is illustrated in Table III. Assume the following conditions: long horizontal and vertical fins (Type B, corresponding to the horizontal fins of Gulf model Oct. 3, 4 wind tunnel tests), weight 1000 lbs.; altitude 20,000 ft.; plane speed 220 ft./sec.; $\delta_E = 0^\circ$; 0 - 15 secs. $\delta_R = 0$, 15 sec. - end $\delta_R = +20^\circ$. From the Oct. 3, 4 wind tunnel data (Gulf Report Fig. 27 or M. I. T. Report Fig. 16) for $\delta_R = 20^\circ$ $\psi = 15.5^\circ$. From Fig. 28 of the Gulf Report or Fig. 17 of the M. I. T. Report, the lift and drag coefficients are obtained for $\psi = 0$ and $\psi = 15.5^\circ$. The side force coefficient C_S is represented as C_{Ly} or as C_L on these figures. From these data the ballistic coefficient curves $\frac{C_D'}{m}$ and $\frac{C_S'}{m}$ are plotted against altitude in the same way as for the "range only" calculation.

[redacted]

In starting the computation, the values of X, Y, Z, V_x, V_y, V_z are all known at t = 0 and can be filled in. In the example of Table III, many of the columns are zero because the initial rudder setting was zero. Values of (A+C), (F-D) and (G+H) are obtained for t = 0. Start the t = 1 calculation by using for $\overline{(A+C)}$, $\overline{(F-D)}$ and $\overline{(G+H)}$ the values of (A+C), (F-D) and (G+H) computed for t = 0. At the end of this line, we will have values of (A+C), (F-D) and (G+H) for t = 1. It can then be seen whether the true average values between t = 0 and t = 1 differ appreciably from the approximate values which have been used for $\overline{(A+C)}$, $\overline{(F-D)}$ and $\overline{(G+H)}$ in making the t = 1 computation. If the error is appreciable, the line should be repeated. Beginning with the t = 2 line, coefficients are available for t = 0 and t = 1 from which to extrapolate the correct average values. At t = 15 sec., the rudder changes to δ_R = + 20. For the t = 15 line, the first sixteen columns are filled in as before, the remaining columns are filled in for the new conditions since these are preparatory for the t = 16 calculation. The result of the calculation illustrated by Table III is shown as the point for t = 21.5 secs. on the Y deflection curve of Fig. 11.

[redacted]

Case III. Combined Pitch and Yaw.

It is assumed that the roll orientation of the bomb is held by the directional gyro so as to maintain the raference plane (plane of horizontal fins) normal to the XZ plane. Combined pitch and yaw will be produced by the simultaneous deflection of elevators and rudders. The aerodynamic force can be resolved into a drag component D parallel to V, and a crosswind component perpendicular to V. The crosswind component can be further resolved into a lift force L in the plane of the vertical fins normal to V, and a side force S in the plana of the horizontal fins normal to V.

A thorough investigation of the problem of computing trajectories for the case of combined pitch and yaw was initiated in November, 1942. A few preliminary results maka it look at least probable that the best method would be to discard the conventional drag, lift and side components and work with the aerodynamic force resolved into a set of components which best fit the simplest set of coordinates to be used in the trajectory calculations. By a proper choice of coordinates, it should be possible to bury a good deal of the difficult computation in the tabulation of the wind tunnel data, rather than going through it each time in the computation of a three-dimensional trajectory. Due to the termination of this work at M. I. T., the consideration of this general problem has been stopped.

[redacted]

Practically no wind tunnel data are available for combined pitch and yaw, and hence only approximate trajectory calculations can be made. From the summary of available data (M.I.T. Report VII p. 15; Gulf Report Oct. 15, 1942, p. 23), the following conclusions can be drawn.

1. The controllability curves for combined pitch and yaw remain essentially the same as for pitch or yaw alone.

2. At most the lift force at trim is decreased 15 per cent when the bomb is simultaneously yawed.

3. For combined pitch and yaw the drag force is roughly that due to either one alone plus the increase in the other over the zero angle value.

Based upon the above facts, an approximate method for computing combined pitch and yaw trajectories can be worked out to use wind tunnel data for pitch or yaw only. Assume the angles of pitch and yaw are given independently from the usual controllability curves. Assume the lift force L and side force S are each given by the corresponding values for pitch or yaw alone. Assume the drag force is given by statement 3 above. The magnitudes of L, S, and D are then given in terms of V^2, δ_E, δ_R, C_L, C_S, and C_D.

The remaining problem is to set up the equations by which these forces can be resolved into X, Y, and Z components. The drag force D is along V and hence resolved by the V_x V_y V_z components which are all known at each step of the calculation. The side force S is in the reference plane and hence handled approximately in the same way as in the "yaw only" case. To set up equations for the orientation of L, introduce the following unit vectors.

V^* Velocity direction of bomb.

L^* Direction of lift force L.

R^* Normal to the reference plane.

The orientation conditions are then

$L^* \cdot V^* = 0$ Lift perpendicular to velocity.

$R^* \cdot V^* = \cos(90^\circ + \theta)$ Definition of pitch angle.

$R^*_y = 0$ Gyro orientation condition.

$L^* \cdot R^* = \cos\theta$ Lift in plane of vertical fins.

$|R^*| = |L^*| = |V^*| = 1$

From these equations we obtain

$$L^*_y = V^*_y \tan\theta$$

$$L^*_z = V^*_x - V^{*2}_y \left\{ V^*_z \tan\theta - \frac{V^*_x}{2} \frac{\cos 2\theta}{\cos^2\theta} \right\}$$

$$L^{*2}_x = V^{*2}_z + V^{*2}_y \left\{ V^{*2}_y + V^{*2}_z + 2V^*_x V^*_z \sin\theta - \tan^2\theta \right\} \quad \text{(approx.)}$$

Since θ is always small

$$\left(\frac{L_x}{L}\right)^2 = \left(\frac{V_z}{V}\right)^2 + \left(\frac{V_y}{V}\right)^2 \left\{ \left(\frac{V_y}{V}\right)^2 + \left(\frac{V_z}{V}\right)^2 + 2\left(\frac{V_x}{V}\right)\left(\frac{V_z}{V}\right) \sin\theta \right\}$$

$$\left(\frac{L_y}{L}\right) = \left(\frac{V_y}{V}\right) \tan\theta \tag{19}$$

$$\left(\frac{L_z}{L}\right) = \left(\frac{V_x}{V}\right) - \left(\frac{V_y}{V}\right)^2 \left\{ \left(\frac{V_z}{V}\right) \tan\theta - \frac{V_x}{2} \right\}$$

For the only case in which a computation has been made, it was assumed that only a small amount of rudder would be used and hence V_y is small compared to V. Equations (19) reduce to

$$\frac{L_x}{L} = \frac{V_z}{V}$$

$$\frac{L_y}{L} = 0 \qquad (20)$$

$$\frac{L_z}{L} = -\frac{V_x}{V}$$

The minus sign is introduced in the expression for L_z (as in equations 11) to correspond to positive L taken upward. It is evident from (20) that this approximate treatment of combined pitch and yaw for small rudder action, amounts to taking the lift as due to pitch only and the side force as due to yaw only.

Introducing (20) and (16a) into the general equations (3) and making the approximation for small V_y, $\sqrt{V_x^2 + V_z^2} = V$, we obtain

$$\frac{dV_x}{dt} = -\frac{C_D'}{m} VV_x + \frac{C_L'}{m} VV_z - \frac{C_S'}{m} V_y V_x$$

$$\frac{dV_y}{dt} = -\frac{C_D'}{m} VV_y + \frac{C_S'}{m} V^2 \qquad (21)$$

$$\frac{dV_z}{dt} = g - \frac{C_D'}{m} VV_z - \frac{C_L'}{m} VV_x - \frac{C_S'}{m} V_y V_z$$

From equations (21) a calculation schedule can be set up similar to the schedules used for "range only" and "azimuth only." For trajectories in which the rudder control is small (V_y very small except perhaps in the last second or two) the three coefficients involving V_y can be omitted.

The only combined pitch and yaw trajectory which has been calculated so far using equations 21 is that for Gulf Bomb No. 9 of the Dec., 1942, Eglin Field tests. The XZ projection of this trajectory is shown in Fig. 4, and the XY projection in Fig. 5.

[redacted]

Projected Ground Position.

The projected ground position of a falling bomb is the point of intersection with the ground of the line of sight from the observer in the plane, to the bomb, to the ground. It is primarily the projected ground position, rather than the true coordinates of the bomb, which is recorded by moving pictures taken from the plane, or which is seen by the observer in the plane. The simplest case to consider is that in which the plane continues to fly a straight horizontal course at constant velocity after releasing the bomb.

Let A = Altitude of plane

$X_p = V_p \times t$ = X coordinate of plane

X_B, Y_B, Z_B are the coordinates of the falling bomb. The projected ground position is then given by the coordinates X_{PG}, Y_{PG} measured from a point on the ground directly under the release point.

$$X_{PG} = X_P - \frac{A}{Z_B}(X_P - X_B)$$
$$Y_{PG} = \frac{A}{Z_B} Y_B \qquad (22)$$

Examples of computed projected ground positions are given by Figs. 6, 7, 8, and 12.

[redacted]

II. COMPUTED TRAJECTORIES FOR EGLIN FIELD TESTS OF DECEMBER, 1942.

Trajectories were computed for nine Gulf bombs assuming the following schedules.

SCHEDULE AND RESULTS FOR NINE GULF BOMBS 1000 LB. WEIGHT FROM ALTITUDE 15,000 FT. AT 150 MI./HR.

Range Control Only. Medium long horizontal, short vertical fins.

No. 1	0-22	$\delta_E = -10°$ (Dive)	T = 30.8 sec.
	22-26	$\delta_E = 0$	X = 4120 ft.
	26-End	$\delta_E = -10°$	
No. 2	0-21	$\delta_E = -20°$ (Dive)	T = 31.2 sec.
	21-25	$\delta_E = 0$	X = 2810 ft.
	25-End	$\delta_E = -20°$	

Azimuth Control Only. Short horizontal and short vertical fins.

No. 3	0-14	$\delta_R = 0$	
	14-21	$\delta_R = +10°$	T = 31.7 sec.
	21-25	$\delta_R = 0$	X = 6467 ft.
	25-End	$\delta_R = +10°$	Y = 976 ft.
No. 4	0-14	$\delta_R = 0$	T = 31.7 sec.
	14-19	$\delta_R = +10°$	X = 6456 ft.
	19-End	$\delta_R = -10°$	Y = -490 ft.

Ears Test. Medium long horizontal, short vertical fins.

No. 5	0-15	$\delta_E = -10°$ (Dive)	
	15-20	$\delta_E = 0$	T = 31.2 sec.
	20-25	$\delta_E = +10°$	X = 5490 ft.
	25-30	$\delta_E = -20°$	
	30-End	$\delta_E = +20°$	

No. 6 Same as No. 5. Ears out 4".

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[redacted]

Stability Test. Long "Side burns" up to end of cylinder.

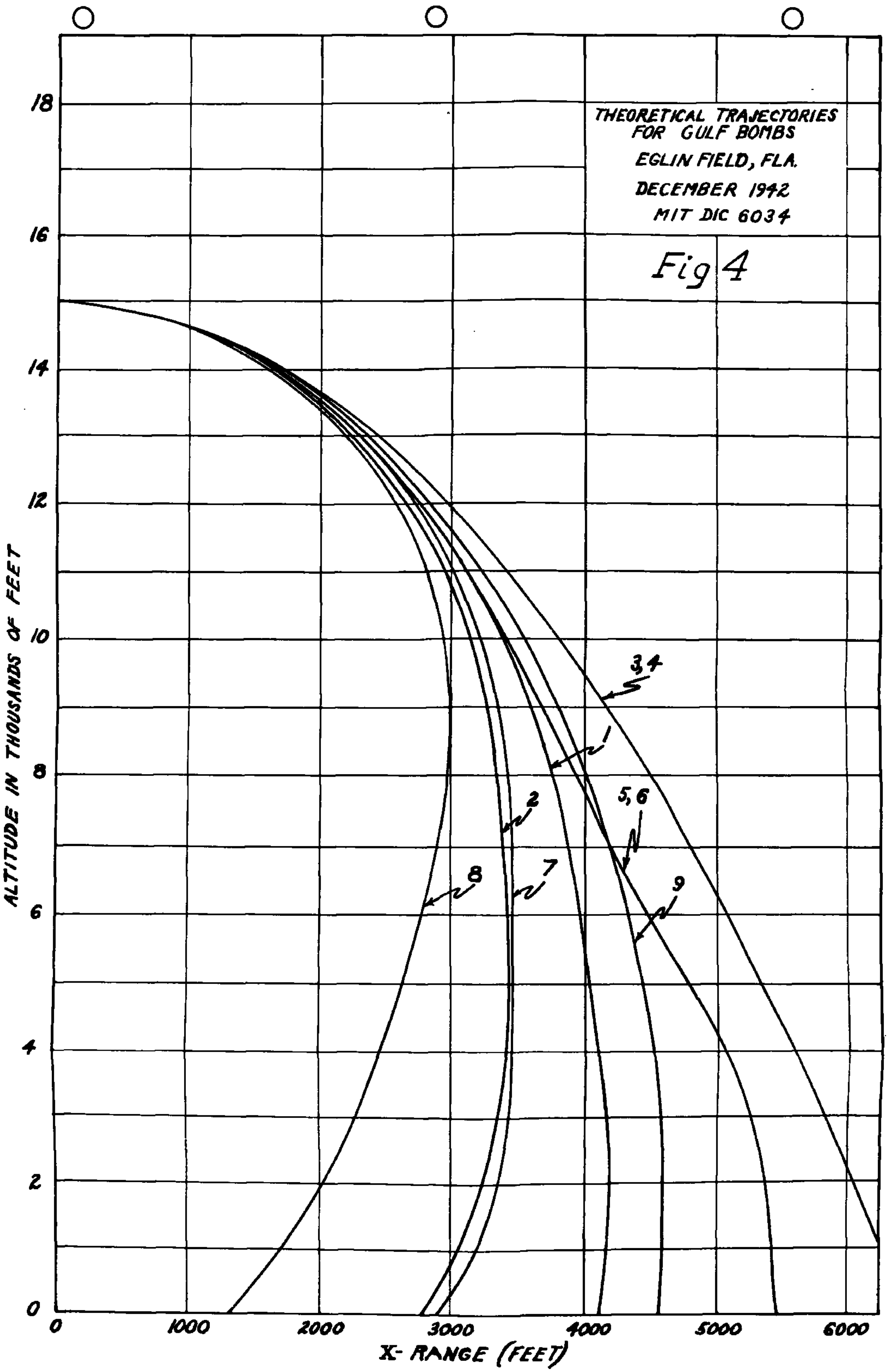
No. 7	Same as No. 1		T = 30.8 sec.
			X = 2980 ft.
No. 8	0-22	$\delta_E = -20°$	T = 30.7 sec.
	22-26	$\delta_E = 0$	X = 1320 ft.
	26-End	$\delta_E = -20°$	

Roll Torque Test. Short horizontal, short vertical fins.

No. 9	0-15	$\delta_E = -10°$	$\delta_R = 0$	T = 31.9 secs.
	15-20	δ "	$\delta_R = +5°$	X = 4561 ft.
	20-25	"	$\delta_R = -5°$	Y = -430 ft.
	25-End	"	$\delta_R = -10°$	

The XZ projections of these nine trajectories are shown in Fig. 4. The XY projections of the trajectories for Nos. 3, 4, and 9 are shown in Fig. 5. The computed projected ground positions for Nos. 3, 4, and 9 are shown in Figs. 6, 7, and 8. Comparison is made with observed curves obtained from moving picture records taken from the plane. The two curves were plotted to the same scale and the calculated curve then fitted to the observed curve in the early part of the flight.

The schedule for bomb No. 7 was changed and the revised form will be designated as No. 7a. Short vertical fins, long "side burn" horizontal fins, weight 1000 lb. elevator $\delta_E = +21°$ throughout flight, $\delta_R = 0$. The computed trajectory is shown in Fig. 9. The experimental curve is obtained from the theodolite records. In the actual trajectory, there were violent roll oscillations in the early part


THEORETICAL TRAJECTORIES FOR GULF BOMBS
EGLIN FIELD, FLA.
DECEMBER 1942
MIT DIC 6034
Fig 4
ALTITUDE IN THOUSANDS OF FEET
X- RANGE (FEET)
0
1000
2000
3000
4000
5000
6000
2
4
6
8
10
12
14
16
18
3,4
1
2
5,6
8
7
9

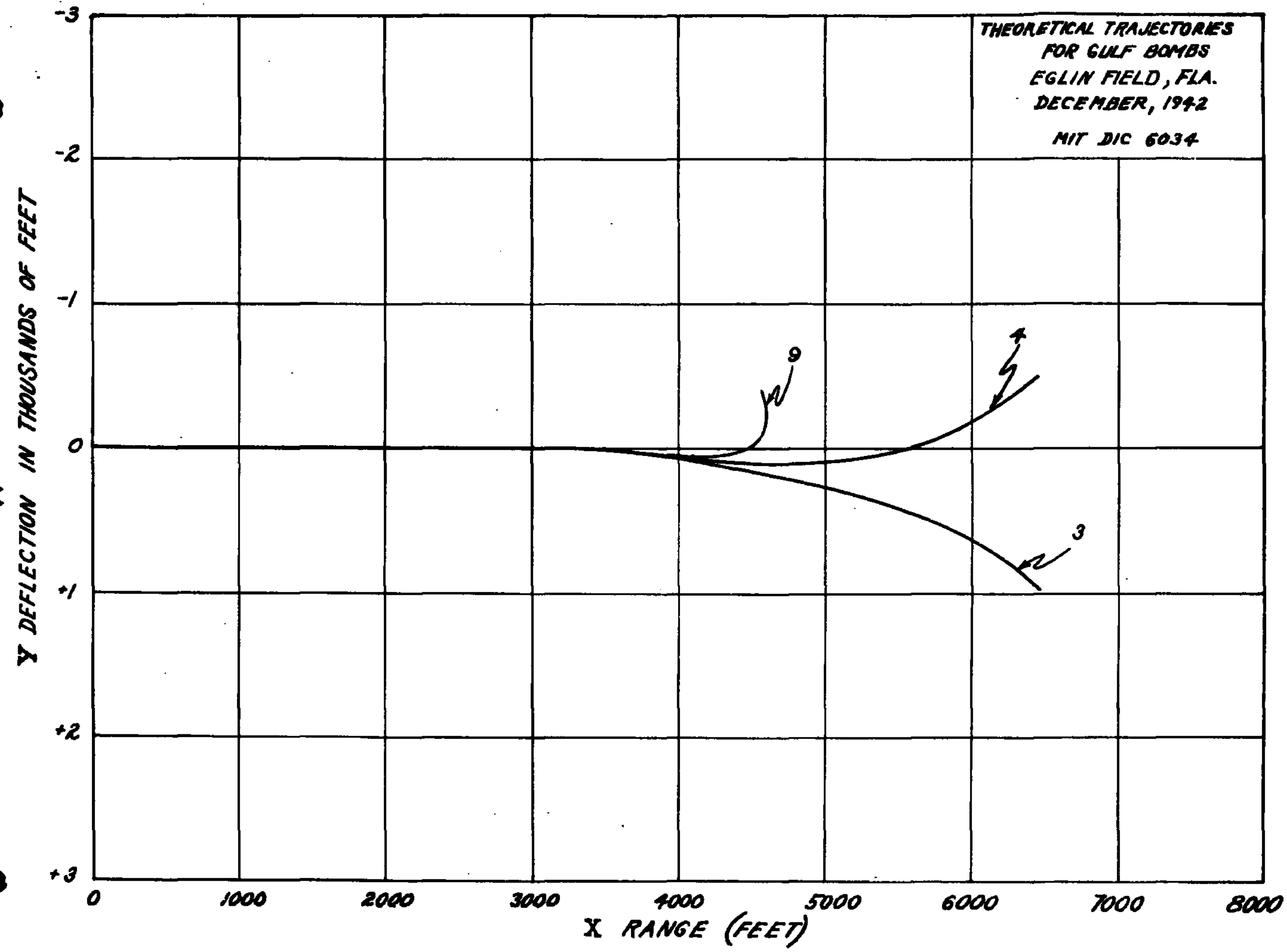

THEORETICAL TRAJECTORIES
FOR GULF BOMBS
EGLIN FIELD, FLA.
DECEMBER, 1942
MIT DIC 6034
Y DEFLECTION IN THOUSANDS OF FEET
-3
-2
-1
0
+1
+2
+3
0
1000
2000
3000
4000
5000
6000
7000
8000
X RANGE (FEET)
9
4
3

Fig.5.

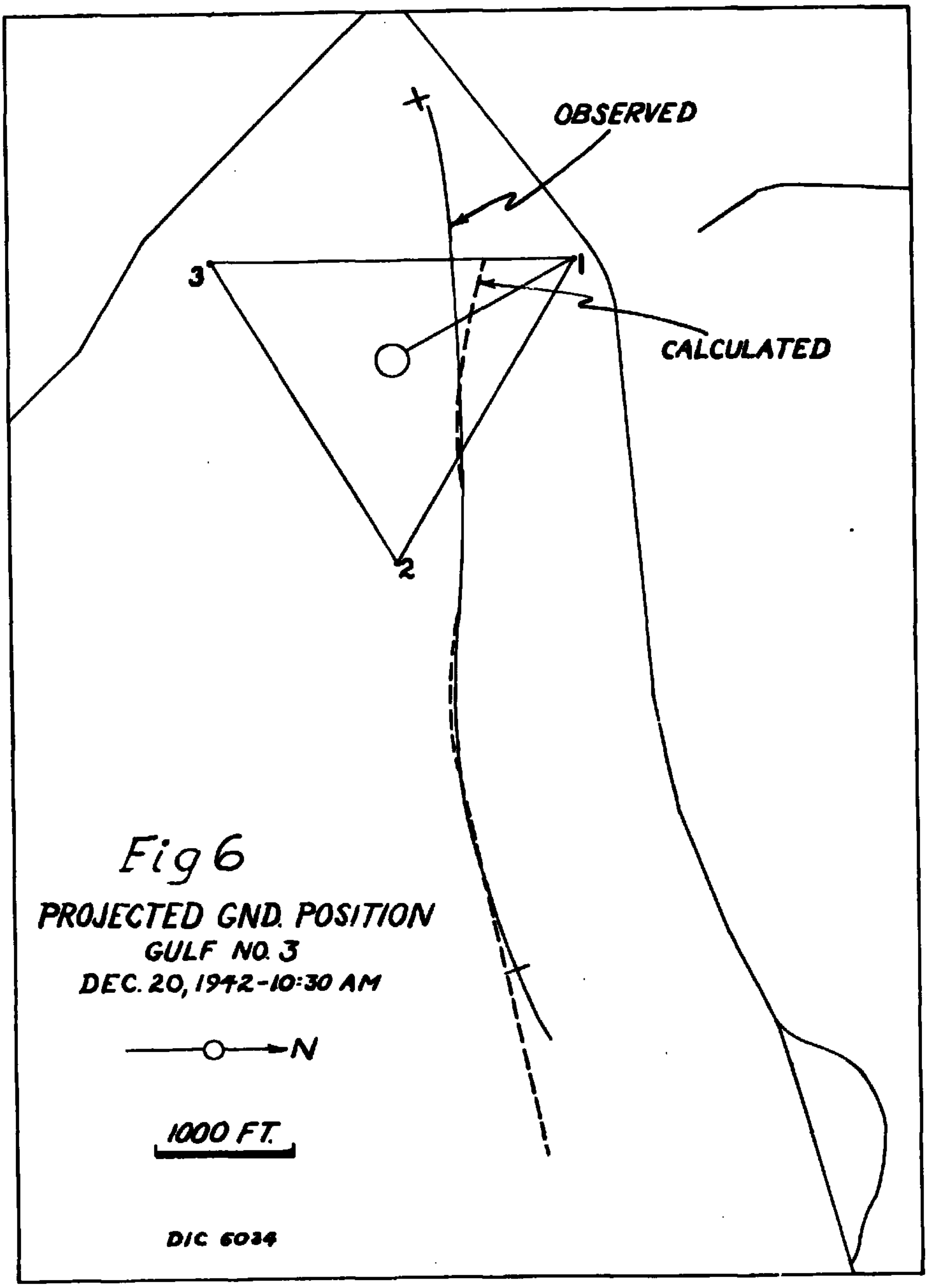


Fig 6
PROJECTED GND. POSITION
GULF NO. 3
DEC. 20, 1942-10:30 AM

DIC 6034

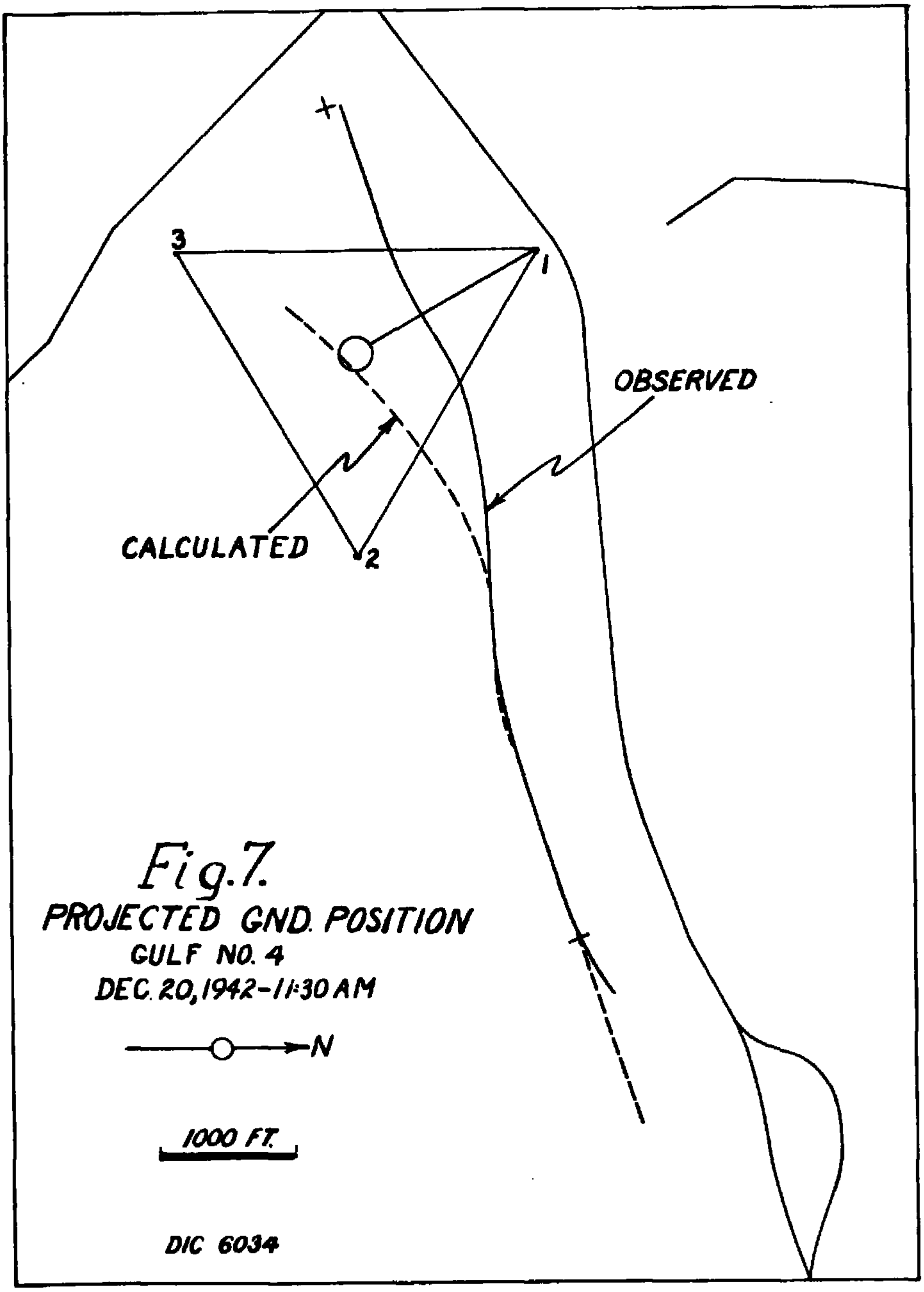


Fig. 7.

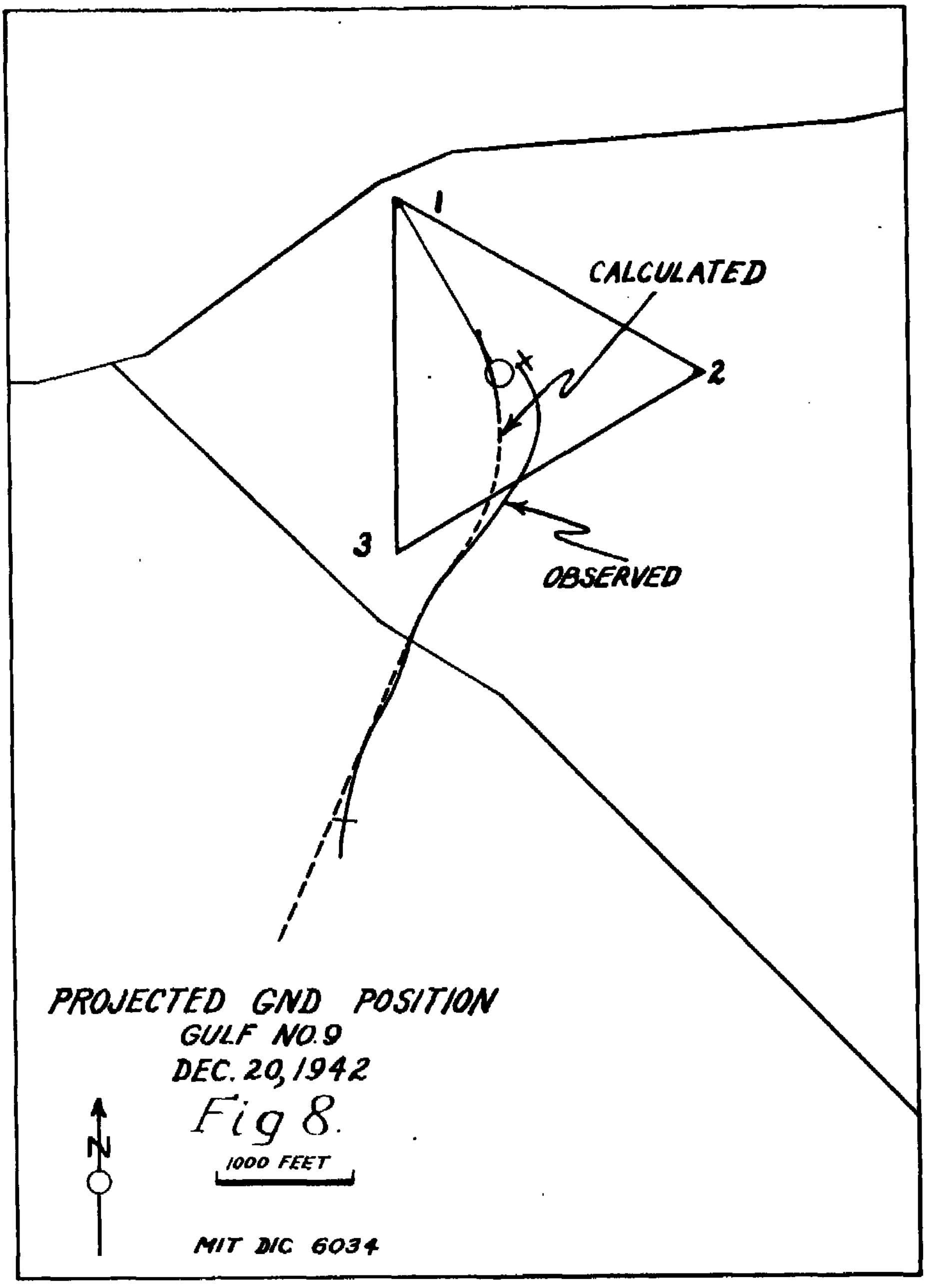
PROJECTED GND. POSITION

GULF NO. 4

DEC. 20, 1942 – 11:30 AM

1000 FT.

DIC 6034



PROJECTED GND POSITION
GULF NO. 9
DEC. 20, 1942
Fig 8.

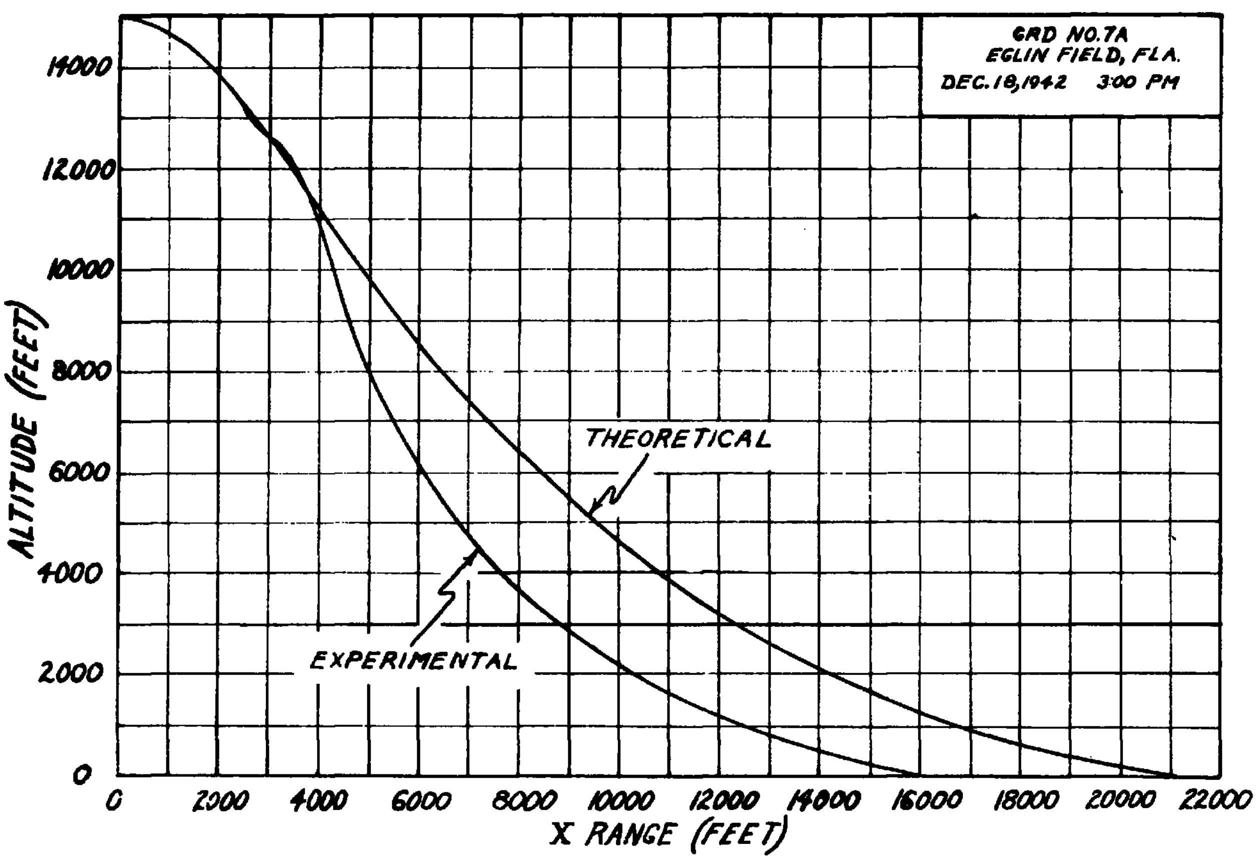


Fig 9.

MIT-DIC 6034

of the flight, the bomb swung through about 90° to the left, and then fortuitously straightened out right side up, and executed a long sail. The experimental curve of Fig. 9 is obtained by folding the two parts of the path into one vertical plane.

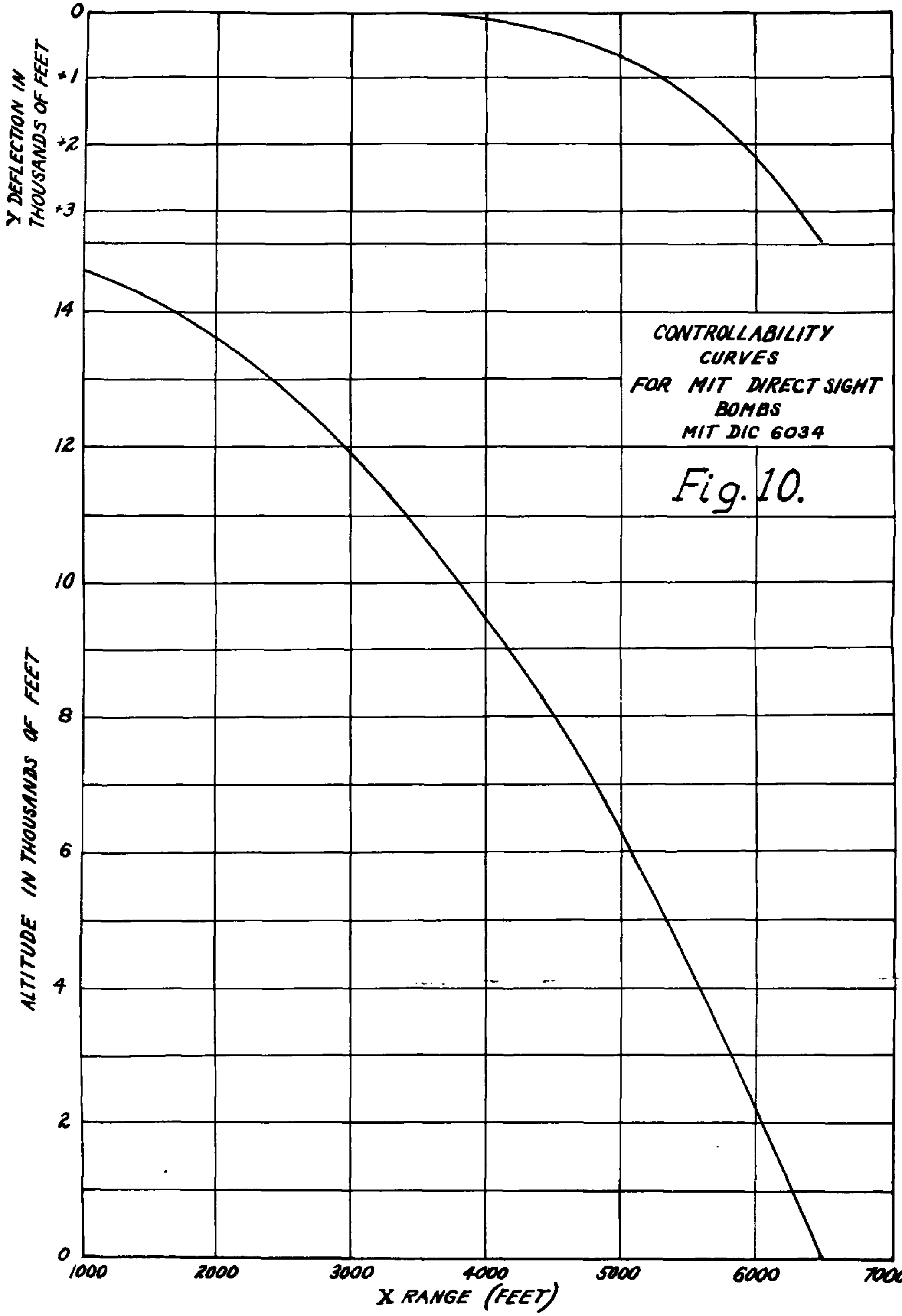
The two M. I. T. Direct Sight Bombs have been described in Progress Report No. 6. The bombs were designed to be steered in azimuth only by on-off rudders (δ_R = 0 or 15°). A trajectory was computed to find the amount of side deflection obtainable for the following schedule

0 - 15 secs. $\delta_R = 0$

15 - End $\delta_R = 15°$

Altitude 15,000 ft., plane speed 150 m.p.h., weight 1000 lbs.

Fig. 10 shows the XZ projection, and the XY projection of this trajectory. As seen from the XY projection, a continuous 15° rudder from 15 seconds on, should achieve a side deflection of a little over 3000 ft. This was considered to be more than ample controllability. It was concluded that a side deflection of 1000 ft. was as much as would be needed and that this could be readily obtained in the last 10 seconds of flight.



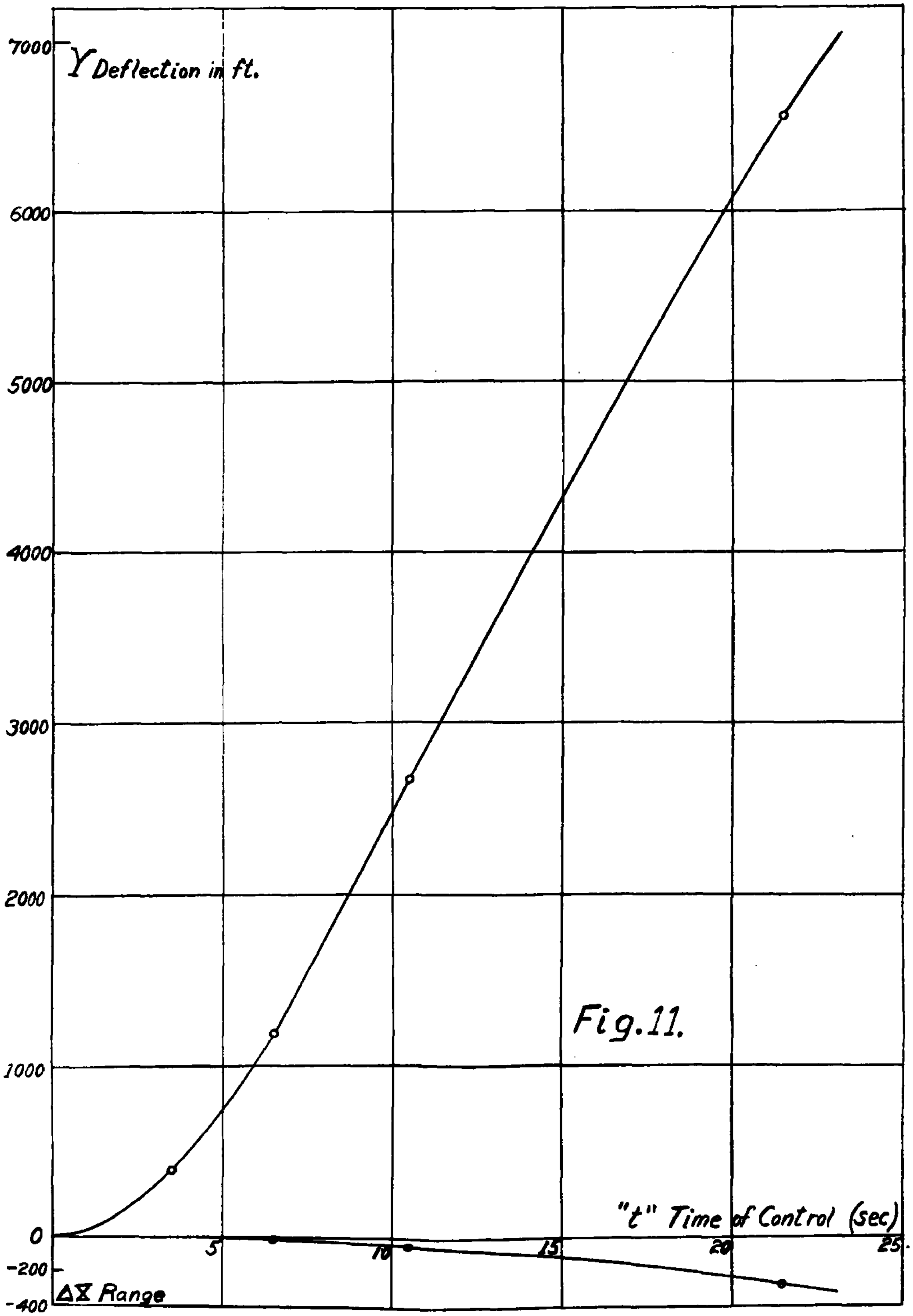
Y DEFLECTION IN THOUSANDS OF FEET
0
+1
+2
+3
ALTITUDE IN THOUSANDS OF FEET
14
12
10
8
6
4
2
0
1000
2000
3000
4000
5000
6000
7000
X RANGE (FEET)
CONTROLLABILITY CURVES FOR MIT DIRECT SIGHT BOMBS
MIT DIC 6034

Fig. 10.

III. CONTROLLABILITY TRAJECTORIES

At the request of H. Spencer, a set of trajectories were computed to find the amount of side deflection which could be produced by full rudder for the last t seconds of flight. Assume the standard Gulf bomb, 1000 lb. weight, dropped from 20,000 ft., with a plane speed of 150 m.p.h. Assume medium long horizontal and vertical fins, using the long fin data of the Oct. 3, 4 wind tunnel measurements. For these conditions, the free fall time is T = 36.49 secs. and the range is X = 7466 ft. Assume that $\delta_R = 0$ until t secs. before 36.49 secs. and is then held at $\delta_R = 20°$ until impact. Complete trajectory calculations were made for five different values of t. Fig. 11 shows the resulting side deflections Y and the shortening in range ΔX plotted against the time of control. From the parabolic shape of the curve near the origin, it is seen that full rudder applied in the last second or two will not produce much of a side deflection, but applied during the last 5 secs. of flight can produce a side deflection of about 700 ft. The important bearing of this point upon different types of rudder control should not be overlooked. When a rudder setting is given, it does not by itself produce either a Y deflection or even a Y component of velocity. All that the rudder setting achieves is a Y component of acceleration. Any desired value of V_y or of Y can then be obtained only by waiting the necessary time.

At the request of the Gulf group, a series of trajectories were computed to find the effect on side deflection



7000
6000
5000
4000
3000
2000
1000
0
-200
-400
Y Deflection in ft.
"t" Time of Control (sec)
5
10
15
20
25
ΔX Range

Fig. 11.

and projected ground position of a 5 second application of full rudder at different parts of the flight. Assume the Gulf bomb of the Oct. 3, 4 wind tunnel tests with short horizontal and vertical fins. Assume the bomb loaded to 1000 lbs., and dropped from 15,000 ft. at a speed of 150 m.p.h. The rudder is to be at zero except during a 5 second period when it is held at $\delta_R = 20°$.

The results of these calculations are shown in Fig. 12. For each control interval there is shown the XY projection labelled "ground trajectory," together with the "projected ground trajectory." The 10-15 sec. interval illustrates how misleading the early part of the projected ground trajectory can be when the control is applied early. The bomb appears to be heading for a point with much greater Y deflection than the final impact point. The later the application of control, the more reliable is the projected ground trajectory in estimating the direction in which the bomb is really heading.

Fig. 13 shows the side deflection produced by a 5 second application of rudder plotted against the average time of application. To a first very rough approximation, a 5 second application of rudder produces the same side deflection regardless of the time of application. The fact that a certain amount of deflection corresponds to a certain number of seconds of control, might have important application in bombing practice, particularly on maneuvering ships.

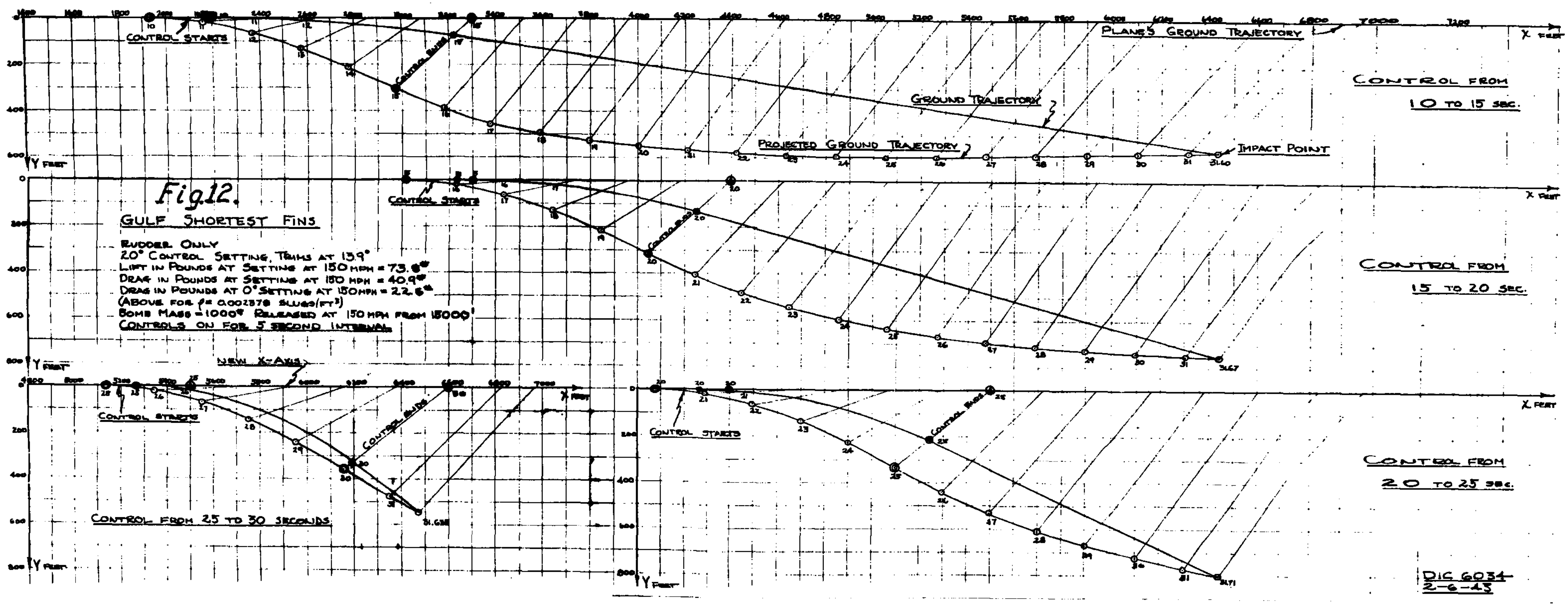


Fig 12.

GULF SHORTEST FINS

Rudder Only
20° Control Setting, Trims at 13.9°
Lift in Pounds at Setting at 150 MPH = 73.8#
Drag in Pounds at Setting at 150 MPH = 40.9#
Drag in Pounds at 0° Setting at 150 MPH = 22.6#
(Above for ρ = 0.002378 slugs/(ft³))
Bomb Mass = 1000# Released at 150 MPH from 15000'
Controls on for 5 second interval

[redacted]

FINAL SIDE DEFLECTION

VS.

AVERAGE TIME DURING WHICH CONTROLS APPLIED

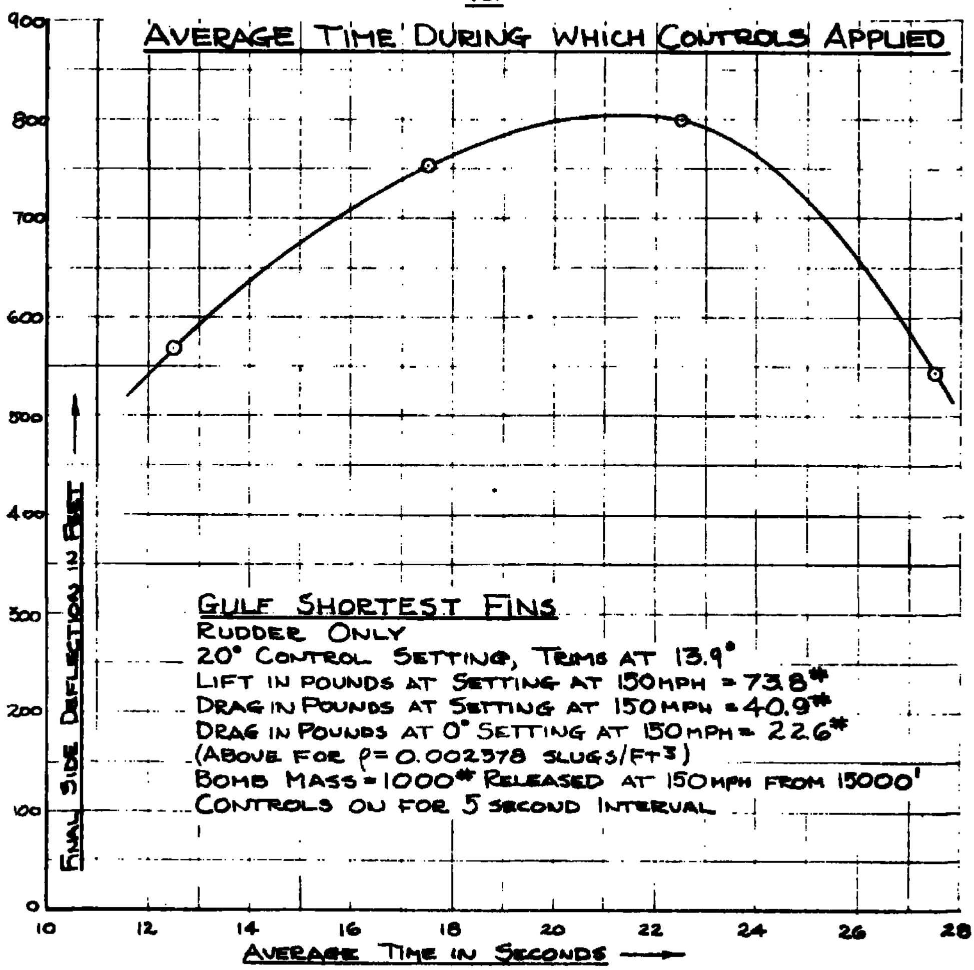


Fig 13

DIC 6034

2-6-43

[redacted]

IV. TRAJECTORY CONSIDERATIONS FOR TELEVISION OR TARGET SEEKING BOMBS.

Prior to the dropping of the M.I.T. Television Bombs in the December, 1942, Eglin Field Tests, a set of trajectory calculations were made to explore the possibility of eliminating ears and mirrors and thus steering the bomb with a television camera mounted rigidly along the bomb axis. The considerations involved are of importance, not only to television operation, but also to the use of target seeking devices.

Represent the longitudinal axis of the bomb by the bomb vector B, the direction in which the bomb is moving by the velocity vector V, and the direction from bomb to target by the target vector T. These vectors are all shown in Fig. 15. With television equipment the angle γ between B and T is seen directly on the receiving screen. With target seeking devices which automatically point themselves toward the target, the device will set itself at an angle γ to the bomb axis. Assuming then a bomb in which γ is observable or automatically recorded, how should γ be varied in order to secure a hit?

In Fig. 15

SECRET

R = Range of target

A = Altitude of dropping

h = A - Z = Height of bomb

$$\left.\begin{aligned} \alpha &= \tan^{-1} \frac{V_x}{V_z} \\ \beta &= \tan^{-1} \frac{R-x}{A-Z} \end{aligned}\right\} \qquad (23)$$

$$\gamma = \alpha - \beta + \theta \qquad (24)$$

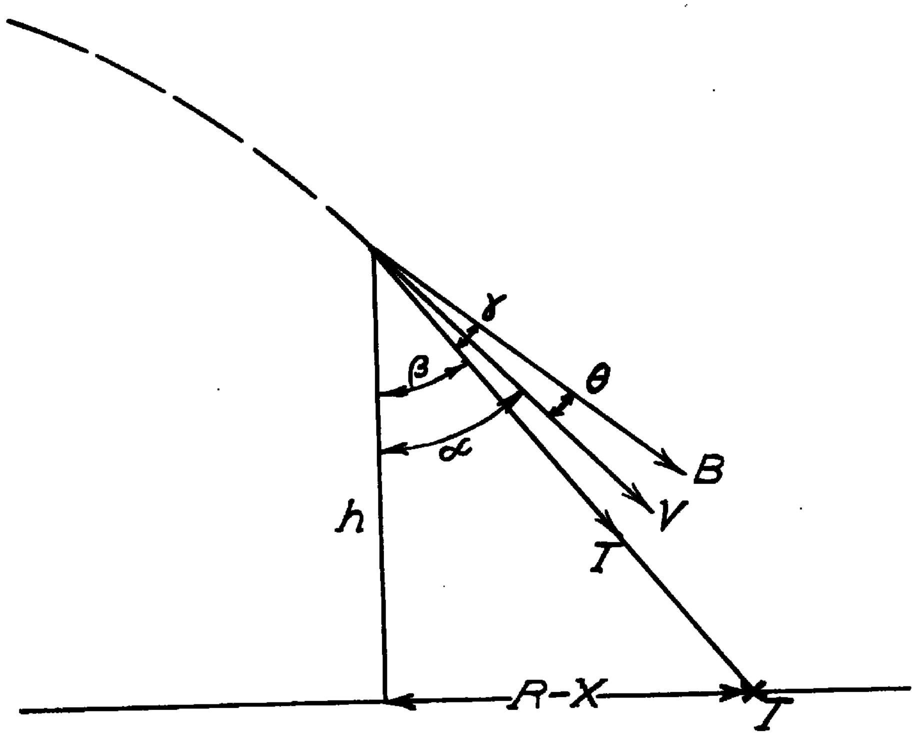


Fig. 15.

Since α and β are determined from (23) at each step of the trajectory calculation, (24) determines the value of pitch angle Θ required to maintain any desired value of γ.

Trajectories were computed for the M. I. T. television bombs described in Progress Report Six. The bomb weight was taken as 500 lbs. (half weight), and the bombs were to be dropped from 15,000 ft. at a speed of 150 m.p.h. Long fin data of the April 13 wind tunnel tests were used.

A free fall trajectory for this bomb ($\delta_E = \delta_R = 0$) was first computed. The trajectory is shown as the curve "Free Fall" on Fig. 14. The computed range for this trajectory is $R = 6132$ ft. The point at $X = 6132$ ft. is then taken as the target for the remaining stationary target trajectories. In the remaining calculations, it is assumed that the bomb falls freely for the first 10 seconds, is then put into a 5° dive until the desired value of γ is secured, and that value of γ is then held until impact.

The simplest procedure would seem to be to point the bomb directly at the target ($\gamma = 0$).

0-10 seconds	$\Theta = 0$
10 sec. - $\gamma = 0$	$\Theta = -5°$
Remainder	$\gamma = 0°$

The trajectory corresponding to this schedule is shown as the curve $\gamma = 0$ on Fig. 14. The impact point is at 6072, so the bomb falls short by 60 ft. This merely confirms previous work which has indicated that steering a bomb by keeping it pointed toward the target will cause it to fall short in range.

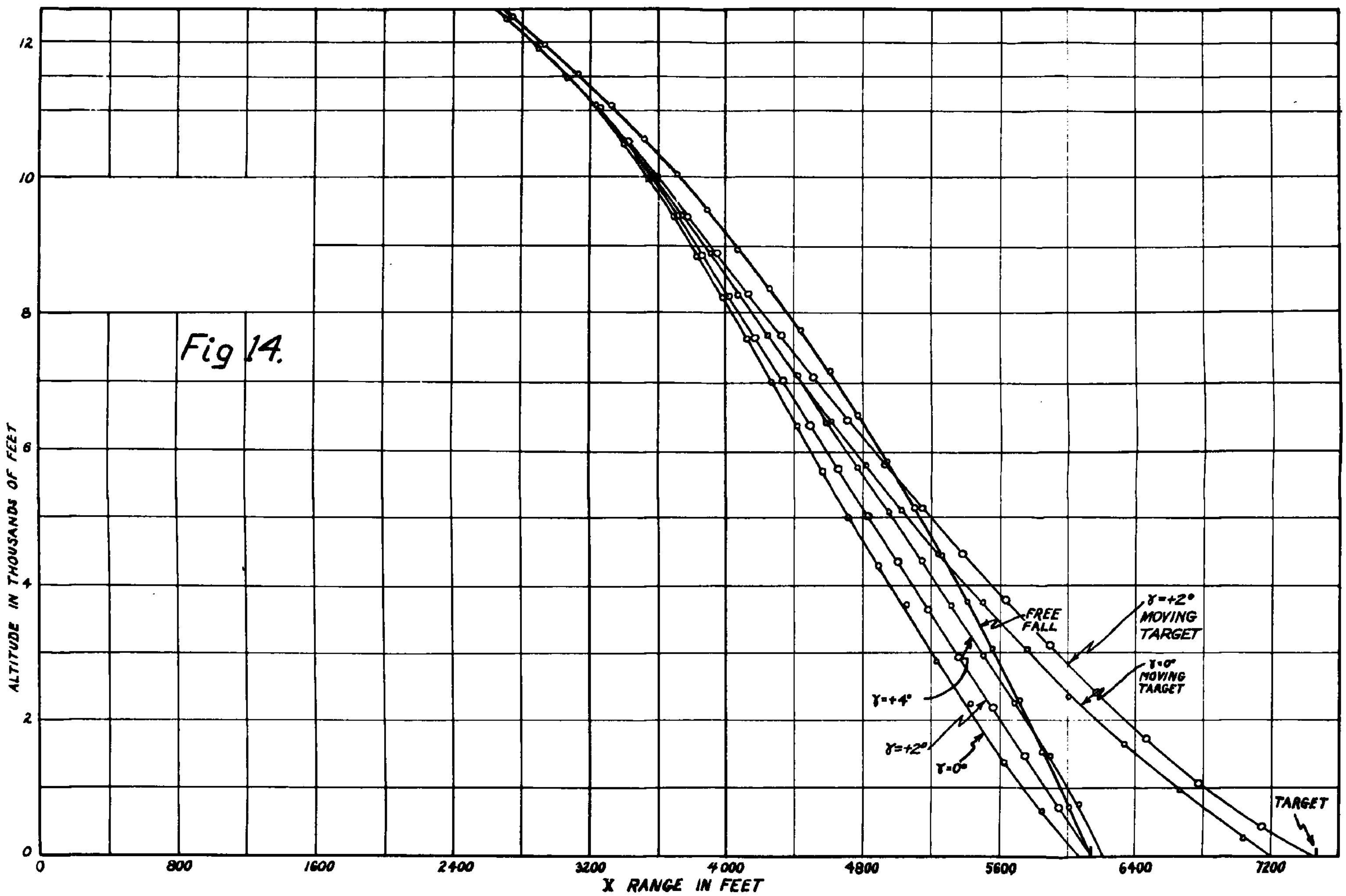


Fig 14.
ALTITUDE IN THOUSANDS OF FEET
X RANGE IN FEET
0
800
1600
2400
3200
4000
4800
5600
6400
7200
2
4
6
8
10
12
FREE FALL
γ=+2° MOVING TARGET
γ=0° MOVING TARGET
γ=+4°
γ=+2°
γ=0°
TARGET

The next calculation was made assuming the bomb to be steered by holding it pointed 2° above the target ($\gamma = +2°$).

0-10 seconds $\quad \theta = 0$

10 secs. - $\gamma = +2°$ $\quad \theta = -5°$

Remainder $\quad \gamma = +2°$

The trajeotory corresponding to this schedule is shown as the curve $\gamma = +2°$ on Fig. 14. The impact point is at 6135 ft., which is only a 3 ft. overshoot and practically a direot hit.

The next calculation was made for $\gamma = +4°$.

0-10 seconds $\quad \theta = 0$

10 secs. - $\gamma = 4°$ $\quad \theta = -5°$

Remainder $\quad \gamma = +4°$

The trajectory is shown as the curve $\gamma = +4°$ on Fig. 14. The impact point is at 6210 ft. The bomb overshoots by 78 ft.

The results of these stationary target calculations indicate that for this bomb load, the component of the weight which tends to make the bomb fall short in range, is adequately compensated by holding $\gamma = 2°$ during the steering. For the bomb carrying the full 1000 lb. load, the angle would be about $\gamma = 4°$. Additional calculations indicate that this value is not very sensitive to the exact schedule in the early part of the flight. If the azimuth correction is to be small, then since there is no component of the weight affecting the azimuth, it is sufficient to point the bomb to the same azimuth as the target. For range the bomb must be pointed above the target by the proper angle γ to allow for the oomponent of weight which affects the range.

[redacted]

The next step was to apply these same considerations to a moving target. The target is assumed to be moving forward at a constant speed of 40 ft. per sec., and to be at the position 6132 ft. at the instant of release. The first calculation was made pointing the bomb at the moving target ($\gamma = 0$).

0-10 secs.	$\theta = 0°$
10 secs. - $\gamma = 0$	$\theta = -5°$
Remainder	$\gamma = 0°$

The trajectory is shown by the curve $\gamma = 0°$ Moving Target on Fig. 14. The bomb strikes at 7220 ft., and at this time the target is at 7468 ft. The bomb falls short by 248 ft.

The next calculation was made for the same moving target, with $\gamma = +2°$.

0-10 seconds	$\theta = 0°$
10 secs. - $\gamma = 2°$	$\theta = -5°$
Remainder	$\gamma = +2°$

The trajectory is shown by the curve $\gamma = +2°$ Moving Target on Fig. 14. The bomb strikes at 7441 ft. and at this time the target is at 7479 ft. Although the bomb falls short by 38 ft., this is practically a direct hit, and represents a very good correction for the 1347 ft. that the target has moved since release of the bomb.

Since the correct value of γ ($\gamma = 2°$) has come out to be the same for both the stationary and the moving targets, it is evident that the correct value does not depend very much upon the motion of the target or the exact schedule in the early part of the flight. For a given bomb, and a given

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approximate trajectory there is a value of the angle γ which if maintained during say the latter half of the flight will properly correct the trajectory in range. It seems very likely that direct application could be made of this principle in television and target seeking bombs.

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Ninth Progress Report:

Written and submitted by:

B. E. Warren

April 30, 1943

SECRET

[redacted]

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2075

TSDIN FORM 69 (13 MAR 47)

Warren, B. E.

AUTHOR(S)

DIVISION: Guided Missiles (1) /2

SECTION: Performance (11) /

CROSS REFERENCES: Bombs - Flight path (16700); Flight path - Measurement (37950)

ATI- 2075

ORIG. AGENCY NUMBER

PR 9

REVISION

U

AMER. TITLE: Ninth progress report on the development of a dirigible bomb

FORG'N. TITLE:

ORIGINATING AGENCY: Massachusetts Inst. of Technology, Div. of Industrial Cooperation

TRANSLATION:

COUNTRY	LANGUAGE	FORG'N.CLASS	U. S.CLASS.	DATE	PAGES	ILLUS.	FEATURES
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1/19/13 + Guided Bombs, Bomb Trajectories

T-2, HQ., AIR MATERIEL COMMAND AIR TECHNICAL INDEX WRIGHT FIELD, OHIO, USAAF

WF-O-21 MAR 47 30M

TSDIN FORM C (13 NO 47)

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Warren, B. E.	DIVISION: Guided Missiles (1) SECTION: Performance (11) CROSS REFERENCES: Bombs - Flight path (16700); Flight path - Measurement (37950)	ORIG. AGENCY NUMBER: PR 9 REVISION

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